

Non-congruence of liquid-gas phase transition of asymmetric nuclear matter

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Abstract.

We first explore the liquid-gas mixed phase in a bulk calculation, where two phases coexist without the geometrical structures. In the case of symmetric nuclear matter, the system behaves congruently, and the Maxwell construction becomes relevant. For asymmetric nuclear matter, on the other hand, the phase equilibrium is no more attained by the Maxwell construction since the liquid and gas phases are non-congruent; the particle fractions become completely different with each other. One of the origins of such non-congruence is attributed to the large symmetry energy.

Subsequently we explore the charge-neutral nuclear matter with electrons by fully applying the Gibbs conditions to figure out the geometrical (pasta) structures in the liquid-gas mixed phase. We emphasize the effects of the surface tension and the Coulomb interaction on the pasta structures. We also discuss the thermal effects on the pasta structures.

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INTRODUCTION

The liquid-gas (LG) phase transition and its relevant equation of state (EOS) is one of the most important issues in nuclear physics and astrophysics. In the collapsing stage of supernovae and the crust region of compact stars, low-density nuclear matter with non-uniform structures has been expected. Such structured matter can be regarded as a mixed phase during the LG phase transition. The mixed phase in thermal equilibrium is often obtained by simply applying the Maxwell construction or more carefully by imposing the Gibbs conditions. However, the Maxwell construction can be used in the case of congruent transition [1], i.e., the coexisting two phases have the same particle fractions with different total densities. A familiar example is the isotherm of boiling water which EOS exhibits a constant pressure in the mixed phase.

On the other hand, for asymmetric nuclear matter or charge-neutral nuclear matter which consists of proton, neutron and electron, the phase transition is generally non-congruent. The particle fractions take different values in each phase. The Gibbs conditions (equilibrium of chemical potentials, temperature and pressure between two phases) then give EOS different from that of the Maxwell construction.

BULK CALCULATION WITH THE RELATIVISTIC MEAN-FIELD MODEL

First, we study the phase coexistence in low-density nuclear matter without considering any geometrical structure. We use the relativistic mean-field (RMF) model for the interaction. In the RMF model, baryons interact via exchange of σ , ω and ρ mesons. All the fields of baryons and mesons are introduced in a Lorentz-invariant way. It is not only relatively simple for numerical calculations, but also sufficiently realistic to reproduce bulk properties of finite nuclei as well as the saturation properties of nuclear matter [2, 3]. One characteristic of our framework is that the Coulomb interaction is properly included in the equations of motion for nucleons and electrons and for meson mean fields. Thus the baryon and electron density profiles, as well as the meson mean fields, are determined in a fully self-consistent way with the Coulomb interaction.

The equilibrium configuration of the liquid (L) and gas (G) phases is obtained by solving the Gibbs conditions as $T^G = T^L$, $P^G = P^L$, $\mu_i^G = \mu_i^L (i = n, p)$, where μ_i denotes the chemical potential of ingredient i . for given temperature T and baryon partial pressure P_B , we can find generally two values of Y_p to satisfy the Gibbs conditions. However, there are the maximum and minimum values of P_B which can give two values of Y_p . Consequently, changing

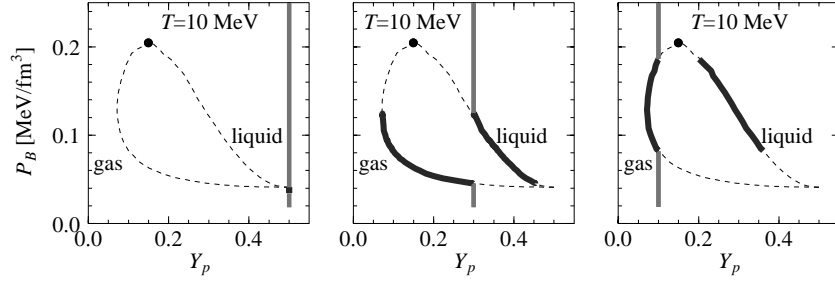


FIGURE 1. Binodal curves (dashed curves) in Y_p - P_B plane at $T = 10$ MeV. Filled circles show the critical points. Vertical lines and thick curves on the binodal curves indicate trajectories of isothermal compression.

the pressure we can draw a closed curve (binodal curve) by connecting these values in Y_p - P_B plane. In Fig. 1, the dashed line shows the phase-coexistence (binodal) curves at $T = 10$ MeV. Density at the region right from the binodal curve (most symmetric) is generally higher than the left (neutron rich) region due to the symmetry energy. Therefore we call the left region a gas and the right region liquid. The inner region of binodal curve is thermodynamically forbidden region. If we put asymmetric nuclear matter with $Y_p = 0.3$ with very low pressure (i.e., a gas) and start to compress it, the trajectory in the Y_p - P_B plane looks as a vertical line in the central panel. At some point, the line encounters the binodal curve. As the system cannot enter the inner region, the system splits into two phases with the same P_B and different Y_p , which corresponds the appearance of mixed phase. When we continue compression of the system, there appears the phase separation of gas and liquid with the various volume fraction like an amorphous state. Eventually Y_p of the right curve equals to that of the total system $Y_p = 0.3$, which means that the phase on the right side, i.e. liquid, dominates the whole system. After that the compression leads the vertical line to go upward. The above change represents a *non-congruent* phase transition from the gas to liquid phase.

In the case of symmetric nuclear matter $Y_p = 0.5$, which is shown in the left panel, the vertical line encounters the binodal curve at one point. Therefore the phase transition from gas to liquid occurs with a constant value of $Y_p = 0.5$. This means the phase transition is *congruent*. The pressure is also constant during the phase transition, which can be obtained by the Maxwell construction. In the case of $Y_p = 0.1$ (neutron-rich), the behavior at lower pressure is the same as the above two cases. However, it should be interesting to see the curve on the left side encounters the value of the total system 0.1 twice, which means the transition proceeds from gas to the mixed phase and back to gas again. This is called “retrograde condensation”, which was observed in the mixture of organic solvent [4]. Note that the gas state at large pressure is not a simple gas but the supercritical one.

The left panel of Fig. 2 shows the binodal curves for various temperatures. At low temperatures, the forbidden region surrounded by the binodal curve is wide. Below 4 MeV, the left part of the binodal curve touches to the origin of the vertical axis $Y_p = 0$, which means the gas phase is vacant or pure neutron matter. At higher temperatures, the region surrounded by the binodal curve becomes narrow, which indicates the enhancement of the congruence.

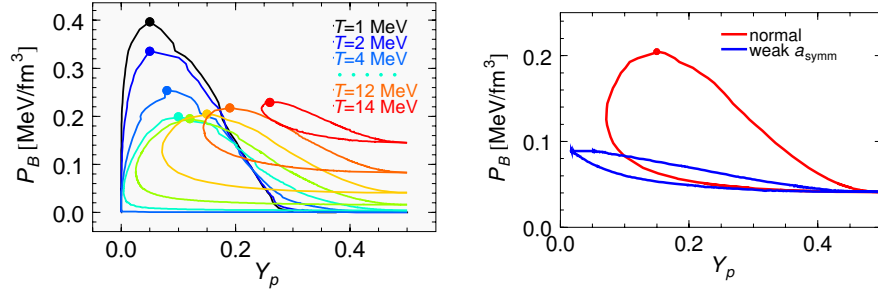


FIGURE 2. Temperature dependence (left) and symmetry-energy dependence (right) of the binodal curves in Y_p - P_B plane. Filled circles show the critical points.

The non-congruence of asymmetric nuclear matter is coming from the strong attraction between proton and neutron (symmetry potential) which makes the particle fraction of the liquid phase more symmetric than the total system. In fact, if we employ a weak symmetry potential in the bulk calculation, the binodal curve will collapse and the

congruence will be enhanced (see the right panel of Fig. 2).

PROPERTY OF NUCLEAR MATTER WITH THE PASTA STRUCTURES

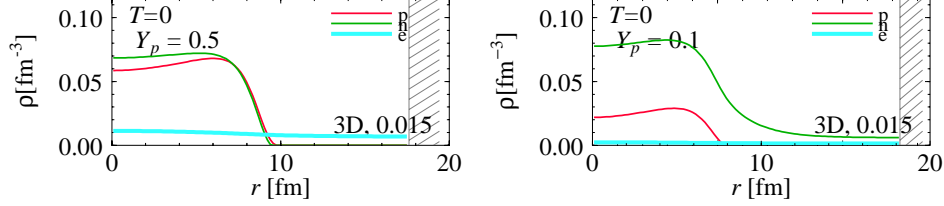


FIGURE 3. Density profiles of symmetric (left) and asymmetric (right) nuclear matter.

Next let us discuss the property of inhomogeneous nuclear matter. It is expected that inhomogeneous nuclear matter exhibits so-called the pasta structures at subnuclear densities [5, 6]. The origin of the pasta structures comes from the balance between the Coulomb energy and the surface energy of matter which are dependent on the geometry such as droplet, rod, slab, tube, bubble, and uniform. To obtain matter with the pasta structures, under the local-density approximation, we solve the equations of motion for the meson mean-fields which are dependent on the position, and the relation between chemical potentials and fermion densities. Details are explained in Refs. [2, 3, 7].

Some examples of the density profiles (droplets) are shown in Fig. 3. In the case of symmetric nuclear matter, density profiles of proton and neutron are almost equal, showing the congruence of the phase transition. On the other hand, asymmetric nuclear matter is non-congruent, accompanying the phase separation with different proton fractions in the liquid and gas phases.

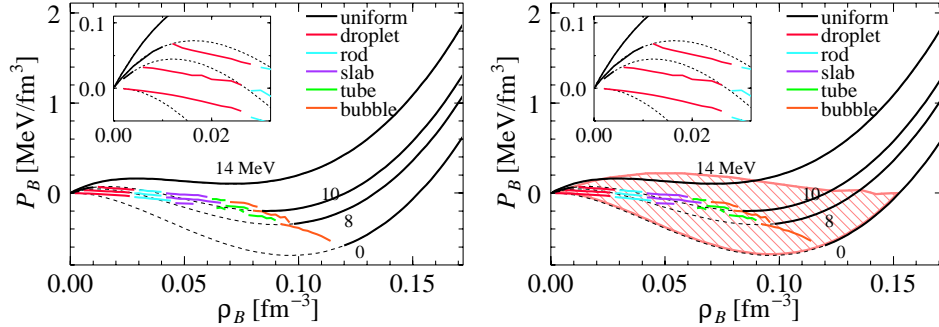


FIGURE 4. Left: EOS of symmetric matter with the pasta structures. Right: Hatched area shows the binodal region.

The resulting EOS of symmetric nuclear matter at various temperatures are presented in the left panel of Fig. 4. Colored curves indicate appearance of various pasta structures while smooth black curves uniform matter for comparison. In the right panel indicated with hatch is the binodal region. We can see that the region where the pasta structures appear is inside the binodal region. Near the upper (higher density) boundary of the binodal region, the appearance of the pasta structures is suppressed by the Coulomb and surface energies which are characteristic of the structured mixed phase.

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